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Research Article

New Approaches on the Theory of Planetary Sciences: Applications of Non-Classical Equations of Mathematical Physics for Plasma Models of Jupiter's Magnetosphere

Mahammad A. Nurmammadov*

Department of Theoretical Astrophysics and Cosmology, Shamakhy Astrophysical Observatory after N. Tusi of the Ministry of Sciences and Education of Republic Azerbaijan, Shamakhy, Azerbaijan

Abstract

In this paper, considered non-classical equations of mathematical physics are applied in the fields of astronomy and astrophysics in the case of plasma models of Jupiter's magnetosphere. It is known that non-classical equations of mathematical physics have applications in gas dynamics, aerodynamics, hydrodynamics, and magneto-hydrodynamics. According to comparisons and observation results of Pioner-10, 11, and Voyager 1-2, considered mathematical models of Jupiter's magnetosphere, which is cold plasma, as searches of Jupiter's Io. At first, the mathematical justification of the physical process of Io concerning plasma was described by a non-classical equation of the Keldysh type. For this reason, using MHD equations for the derivation of the model equations of cold plasma and hot plasma on Jupiter's magnetosphere. In the region tail of Jupiter given analyses of basic model equations of the Jupiter magnetosphere for the equilibrium between magnetic force, pressure gradient, and centrifugal force in the presence of plasma rotations. Additionally, based on the basic theoretical and observational results, the role of the Alfven Mach number with a constant Euler potential parameter in the region tail of Jupiter's magnetosphere proves the justification of the steady magneto-hydrodynamic equilibrium. as agreed previously in the results of observation Voyager 1,2. Therefore, in the magnetosphere, Jupiter's hot and cold plasma describe the same class equation of Keldysh-Tricomi types. In this case, the exact solution is obtained by integrals, which are first expressed as analytical formulas. Theoretical aspects of the model hot and cold plasma on the tail magnetosphere contain concepts of reconnection, which connects lost mass from Jupiter's Io. Such an effect reconnection coronal problem as Parker's also occurs by lost temperature and energy dissipation. Lorentz force, supported by means of solar wind, changes cold plasma to hot plasma in cases where a magnetic disk acts as a balancing mechanical equilibrium to retain cold-hot plasma. For motivation, both mathematical and physical, we used some figures, a table, and an appendix. Note that considered approaches to the theory of planetary sciences at first time applicable for Jupiter.

Keywords

Non-Classical Models, Jupiter's Io, Non-Classical Approaches, Mixed Keldysh Type Equation, Hydro-Dynamical Equilibrium, Planetary, Astrophysics

*Corresponding author: nurmamedov_55@mail.ru (Mahammad A. Nurmammadov)

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1. Introduction

Since our research work is related to non-classical equations of mathematical physics, astrophysical hydrodynamics, plasma, cold plasma, Jupiter's magnetosphere, the satellite Io, mechanical movements, stable magneto-hydrodynamics, as well as aerodynamic and gas-dynamic transient processes such as sound barriers, which are all described in stages with partial differential equations, In this case, it is precisely the mechanical and physical processes of cold plasma from the satellite Io to the Jupiter's magnetosphere plasma that are modeled and described using the so-called non-classical equations of mathematical physics. Therefore, it is appropriate to give some important concepts and small overview comparisons of the above-mentioned terminology.

Non-classical Equations of Mathematical Physics:

Mathematical physics is perhaps the only mathematical science that is based not on axioms (such as algebra) or fundamental concepts (such as calculus), but directly on the laws of nature. Archimedes is considered to be the father of mathematical physics, and in his mathematical research, he widely used the natural science concepts known to him. Some of them (for example, Archimedes' law on the floating of bodies in liquid) are still applied today, and some (for example, the limitation of the universe to the sphere of stars) have long lost their relevance. After approximately two millennia, with the development of sciences, especially physics and mathematics, came the realization of the object of the study of mathematical physics: equations and systems of partial differential equations that model certain natural phenomena. The first step towards such an understanding was made by P. Fermat and J.L.R. D'Alembert, having obtained and studied the first mathematical models—the equation of heat propagation in a thin rod $(U_t = aU_{xx}, \Delta$ -operators Laplace) and the equation of string vibration $(U_{tt} = aU_{xx})$, respectively. Soon, these models were generalized and are now known as the heat equation $(U_t = a\Delta U)$. For quite a long time, these types of equations were sufficient to describe the phenomena of new models of mathematical physics such as, for example, the system of Maxwell's equations, which models the dynamics of the electromagnetic field, or the system of Navier-Stokes equations, which models the flow of a viscous incompressible fluid. A tradition has developed: all mathematical models whose equations can be attributed to one of these three types are called classical. The term "non-classical equations of mathematical physics" was introduced into use by V.N. Vragov [2] and his students in order to isolate the area of his research. But below given strictly justification of definition non-classical equation in mathematical physics.

Definition of a non-classical equation in mathematical physics:

Non-classical models of mathematical physics are those whose representations in the form of equations or systems of partial differential equations do not fit within the framework of one of the classical types: elliptic, parabolic, or hyperbolic. In particular, non-classical models include those described by mixed type equations (for example, the Tricomi equation [3], degenerate equations (for example, the Keldysh equation [4]), or Sobolev type equations (for example, the Barenblatt-Zheltov-Kochina equation [5, 6]. Apparently, non-classical equations of mathematical physics first appeared in the works of S.A. Chapligin [7] in the study of transonic flows, where the so-called mixed-type equations were introduced. In particular, the study of equations of mixed type in connection with the Tricomi problem, transonic gas dynamics ([8, 9]), aerodynamics, magneto hydrodynamic flows with transition through the speed of sound and Alphen, fluid flows in an open channel, with the theory of infinitesimal bending of surfaces, as well as with the momentless theory of shells with curvature of alternating sign, and with many other questions of mechanics. Currently, the scope of non-classical equations of mathematical physics, together with functional analysis of the connection between mechanics, astrophysics, galaxies, black holes, etc., is expanded by the addition of equations of forward-backward ([10-13]).

Note: Astrophysical fluid dynamics is a modern branch of astronomy that includes fluid mechanics, which deals with the movement of fluids such as the gases that make up stars or any fluid that is found in outer space. Based on the point of view of this definition, the author M.A. Nurmammadov (see [13]), combining and expanding the scope of application, noted as in the definition of the equation of non-classical mathematical physics from the mechanical, hydrodynamic, aerodynamic gas-dynamic sense in the terminology of the Mach number in addition, astrophysical hydrodynamics, for the first time, considers new applied problems in the fields of astronomy and astrophysics. The mutual linear transformation of electromagnetic and plasma waves in an inhomogeneous plasma is of considerable interest and has been studied by many (see, for example, [14-19]). However, in all cases, only a plane-layered medium was considered, i.e., a medium whose parameters depend on one spatial coordinate. Since such conditions are never encountered in practice, it is desirable to study the linear transformation process under more realistic assumptions. As is known, the linear transformation of electromagnetic waves into plasma waves in a flat layer is closely related to the field features in cold plasma (see [15, 18, 20]. It is the presence of such features that is a necessary condition for the transformation, and the energy carried away by the plasma wave is equal to the energy absorbed in the cold plasma. This circumstance makes it essentially unnecessary to solve the transformation problem if the simpler problem for cold plasma is solved. A simple physical consideration (see [15-17, 21]), from which the indicated connection between transformation and absorption follows, can be directly generalized to the case of arbitrary inhomogeneity. Therefore, it can be stated that in the general case, the question of the field features in cold plasma is of greatest interest. The model of two-dimensional inhomogeneity covers a large number of plasma configurations, including a toroidal system of the Tokamak type. Among the many equations of mathematical physics that move along a smooth curve from the elliptic to the hyperbolic type, constant attention has been paid only to the equations of transonic flow. In this article, in the introduction section and a brief overview, we consider elliptic-hyperbolic equations arising in a simple model of the propagation of electromagnetic waves through a zero-temperature plasma. Solutions to such equations will likely have significantly less regularity than solutions to linearized transonic flow equations. Recognizing the interdisciplinary nature of the topic, we assume familiarity with physics, but not necessarily plasma physics, and analysis, but not necessarily elliptic-hyperbolic equations. However, physics is limited to a review of fundamental results in the physical sense (the "cold plasma model" [17, 21]), and while mathematical results are somewhat more technical. Since plasma is an ionized gas, plasma simulations use equations of gas dynamics. In this case, elliptic-hyperbolic equations of mixed type arise naturally, and for the first time, an equation of this type appeared in Chaplygin's work [7] on gas jets in 1902. They have analogies for equations arising from another physical problem. By continuing such research in various mechanical, physical, and geometric contexts (see [16-21]), one can hope to eventually obtain a natural theory for linear elliptic-hyperbolic equations in partial derivatives ([26, 27]).

Important explanations cold plasmas and mathematical models.

1.1. Physical Background with Bound Cold Plasmas

The plasma state is characterized by the dominance of long-range nonlinear effects. In this state, it is especially difficult to obtain mathematical problems that can be formulated with a satisfactory degree of rigor and for which the existence of solutions can be demonstrated. Without proof of the existence and uniqueness of solutions, which, in particular, determines the functional spaces in which the solutions lie, it is difficult to set suitable boundary conditions in numerical experiments and assess the reliability of the results obtained. If one hopes to obtain a solvable mathematical problem, it is usually necessary to impose stringent assumptions on both the plasma and the applied field. Perhaps the most severe of these sets the plasma temperature to zero. This allows us to completely neglect the liquid properties of the medium, which is then considered a linear dielectric. Somewhat surprisingly, the zero plasma temperature assumption is a useful first approximation to the products of tokamaks: low-density plasma that is remarkably free from expected high-temperature phenomena such as collisions and wall effects. [28, 29] More generally, the cold plasma model approximates the effects of low-amplitude electromagnetic waves propagating with phase velocities that are quite large compared to the thermal velocity of particles. Note that the term "cold plasma" is very ambiguous. Although we believe that this means zero temperature, in the astrophysical literature, interstellar plasma of order K is usually called "cold" (see, for example, [20]). More recently, an "ultra-cold" neutral plasma has been created experimentally, having an electron temperature of 10^3 K to K and an ion temperature of K to 10^{-3} K. The cold plasma model explored in this paper appears to be too simple to provide a quantitative representation of this plasma. Other physical hypotheses put forward in this review are also quite restrictive: although the plasma is not assumed to be homogeneous, the inhomogeneity is assumed to be two-dimensional, so the governing equations of the model are also essentially two-dimensional. For the most part, the outstanding mathematical problems related to the cold plasma model are boundary value problems for Maxwell's equations. The dielectric tensor for these equations will give them an elliptic type in one part of the domain of definition and a hyperbolic type in the rest, except for a smooth curve (parabolic line) separating the two domains. Little is known about the formulation of correct boundary value problems for equations that change type in such a way (belongs to the class of so-called equations of non-classical mathematical physics M. A. Nurmammadov ([13]), especially since the equations arising in the cold plasma model apparently have some fundamental differences from those that arise in gas dynamics. In this aspect, the conclusions of the plasma model equation differ from the classical MHD equations in that we obtain direct physical nature using the dielectric tensor, which gives a mathematical model of cold and hot plasma. In this case, cold plasma acts as a source of hot plasma in the presence of the solar wind. The advantage of such a single physical basis also gives rise to a single-form model of an elliptic-type equation in one part of the domain of definition. At the same time, the rest of the plasma is of the hyperbolic type, with the exception of the smooth curve (parabolic line) separating the two regions, which for cold and hot plasmas differs only in coefficients. Naturally, since these equations must have solvability, thanks to the method of functional analysis, this necessity is established in the weighted spaces of S. L. Sobolev [1]. This fact was first studied by the author of this article, whose mechanical, hydrodynamic, aerodynamic, and gas-dynamic meanings in the terminology of the Mach number can be found in the author's work [13].

1.2. Mathematical Background with Connected Cold Plasma

The analysis of the Beltrami projective disk model for hyperbolic space is, in a sense, very old mathematics. Beltrami introduced the projection disk in 1868 as one of the earliest Euclidean models of non-Euclidean space (see [28, 29, 20]). But it also arises in the context of some new mathematics related to variational problems in Murkowski space and Hodge theory on pseudo-Riemannian manifolds. The Beltrami model is used as a starting point for a review of aspects of the geometric vibrational theory of mixed Riemannian-Lorentz domains in which the metric signature changes signs along a smooth hypersurface. In variational problems that are created for the existence of harmonic fields on an extended projected disk that is, solutions to the Hodge equations [31] an important factor for physical applications is whether the elliptic-hyperbolic differential operator has a real principal type, in which case the principal symbol of the operator is real. Since the zero bicharacteristic is the integral curve of the Hamiltonian system canonically related to the principal symbol, the basic analytical properties of real operators of the main type depend only on the main symbol and not on the form of the lower terms. If the operator has a real main type, the ideas of microlocal analysis can be applied to construct a natural theory of boundary regularity applicable to the elliptic-hyperbolic case [32]. In recent years, the author M. A. Nurmammadov [33-35] has been studying the generalized form of equations and systems of equations in a multidimensional domain with several degenerate lines (hyperplanes) of mixed hyperbolic-elliptic type M.V. Keldysh. Further, based on the mechanics of continuum, media, liquid and gas, and mathematical points of view, plasma is consistent with the following characteristic states: Plasma is a fluid made up of electrons and one or more types of ions. Since it is a fluid, its evolution must satisfy the equations of hydrodynamics. But because the liquid particles are charged, they act as sources of an electromagnetic field, which is determined by Maxwell's equations. The presence of an intrinsic field results in highly nonlinear behavior. Indeed, the dominance of long-range electromagnetic interactions over short-range interatomic or intermolecular forces is often said to be the defining characteristic of the plasma state. If the plasma has a zero temperature, then as known of Amonton's law implies that the pressure term in the equations of fluid motion will also be equal to zero, and the laws of hydrodynamics will operate only through the laws of conservation of mass and momentum. In fact, since collisions can be neglected, the fluid aspect of the medium can be virtually ignored. Plasma is then thought of as a static dielectric through which electromagnetic waves propagate.

2. Structure of Jupiter's Magnetosphere and Applicable Conditions of Mathematical Models in Plasma

2.1. Structure of the Magnetosphere of Jupiter and the Io Satellite as a Source of Plasmas

Before starting to explain that Jupiter's satellite Io is a source of plasma, to simulate plasma on Jupiter, we will consider the mechanical and physical-chemical processes of the interaction of Io with the magnetosphere of Jupiter, as well as the evolution of the formation of the plasma torus Io. Therefore, it is appropriate to refer to the volcanic process in “Io Volcano Observer: Following the Heat and Hunting Clues to Planet Evolution." Tricia Talbert On March 18, 2021, a proposed mission called the Io Volcano Observer (IVO) would visit Jupiter's moon Io, which is a true volcanic wonderland with hundreds of erupting volcanoes gushing tons of molten lava and sulfurous gases at any moment. NASA's Galileo spacecraft caught Jupiter's moon Io, the planet's third largest moon, experiencing a volcanic eruption. Caught in a perpetual tug-of-war between Jupiter's formidable gravity and the smaller, persistent pulls of neighboring moons, Io's warped orbit causes it to twist as it flies around the gas giant. The stretch causes friction and intense heating inside Io, causing massive eruptions on its surface. NASA's Galileo spacecraft captured the highest resolution images of Jupiter's moon Io in July 1999. This color mosaic uses near-infrared, green, and violet filters to bring closer what the human eye sees. Much of Io's surface is pastel, accented by black, brown, green, orange, and red elements near active volcanic centers. Io is the most active volcanic world in the solar system. Set the clock back a few billion years, and this could be the surface of any young rocky planet. But today, in our solar system, only Io is the site of such hyperactivity. Under the colossal gravitational pull of Jupiter and the passing orbital pulls of sister moons Europa and Ganymede, Io is subject to harsh tides that stretch and compress the Moon as it moves along its elliptical path. Scientists know that these tidal forces generate extreme heat inside Io, resulting in a heat flow 20 times greater than on Earth, and are overall an important planetary process in the universe. But we still have a deep understanding of how they actually work, says Alfred McEwen, a planetary geologist and professor at the University of Arizona's Lunar and Planetary Laboratory. NASA’s Galileo spacecraft caught Jupiter’s moon Io, the planet’s third-largest moon, undergoing a volcanic eruption. Locked in a perpetual tug of war between the imposing gravity of Jupiter and the smaller, consistent pulls of its neighboring moons, Io’s distorted orbit causes it to flex as it swoops around the gas giant. The stretching causes friction and intense heat in Io’s interior, sparking massive eruptions across its surface.

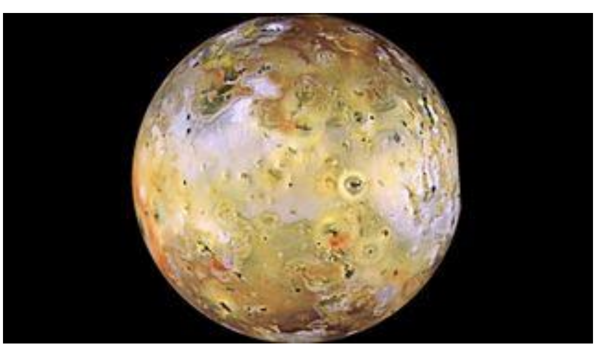

Figure 1. *Jupiter’s moon Io is the most volcanically active world in the solar system. This high-resolution image of Jupiter’s fifth moon was captured by NASA’s Galileo spacecraft and was published on 18, Dec. 1997. (Image credit: NASA/JPL/University of Arizona) NASA’s Juno to Get Close Look at Jupiter’s Volcanic Moon Io on Dec. 30.*

2.2. The Jupiter’s Magnetosphere and Some Important Basic Conditions of Mathematical Models in Plasma

The magnetosphere of Jupiter is a cavity created in the solar wind, the planetary magnetic field of Jupiter, where various processes of interaction between the solar wind, the interplanetary magnetic field, Jupiter’s own magnetic field, and the surrounding plasma occur. The existence of Jupiter's magnetosphere was revealed through radio observations in the late 1950s, first directly observed by “Pioneer- 10” in 1973. Jupiter's internal magnetic field is generated by an electric current flowing in the planet's outer core, which consists of metallic hydrogen (gray layer). At the same time, it is a degenerate state of matter and has some remarkable properties, such as high-temperature superconductivity and a high specific heat of phase transition. So, volcanic eruptions on Jupiter's satellite Io throw a large volume of gray oxide into space, forming a large gas torus around the planet. The torus replenishes the planet's magnetic field with plasma, a partially or fully ionized gas formed from neutral atoms (or molecules) and charged particles (ions and electrons). The most important feature of plasma is its quasi-neutrality, which means that the volume densities of the positive and negative charged particles from which it is formed are almost the same. Plasma is sometimes called the fourth (after solid, liquid, and gas), which, as it rotates, expands into a pancake-like structure known as a magnetic disk. In essence, the magnetosphere of Jupiter is formed by the plasma of Io and its own rotation to a much greater extent than by the solar winds, which are a flow of ionized particles (mainly helium-hydrogen plasma) flowing out of the solar corona at a speed of 300–1200 km/s into the surrounding outer space. The Jovian magnetosphere is a complex structure that includes a bow shock wave, a magnetic transition layer, a magnetopause, a magneto tail, a magnetic disk, and other components. The magnetic field around Jupiter is created due to a number of phenomena, for example, liquid circulation in the planet's core (internal field), electric current in the plasma surrounding Jupiter, and currents flowing at the boundary of the planetary magnetosphere. The magnetosphere is immersed in the solar wind plasma, carrying with it an interplanetary magnetic field (see [36-38]). But while the earth's core is made of molten iron and nickel, Jupiter's core is made of metallic hydrogen. Like Earth's, Jovian's magnetic field is primarily a dipole, with the north and south magnetic poles at opposite ends of the magnetic axes [39]. However, on Jupiter, the north and south magnetic poles of the dipole lie in the hemispheres of the same name on the planet, while in the case of the Earth, on the contrary, the north magnetic pole of the dipole is located in the southern hemisphere and the south in the northern hemisphere [41]. The magnetic field of Jupiter also contains higher multiplicative components (quadrupole, octupole, etc.), but they are at least an order of magnitude weaker than the dipole component. The dipole is tilted approximately 10 °relative to Jupiter's rotation axis; this inclination is close to the Earth's (11.3 °) ([39, 43, 44]). The equatorial magnetic field induction is approximately 428 MkTesla, approximately 10 times greater than the Earth's, which corresponds to a magnetic dipole moment of about 1.53 $\times 10^{20}$Tesla. m^3 (18,000 times that of Earth [36]. Jovian's magnetic field rotates at the same angular speed as the region below the atmosphere, with a period of 9 hours and 55 minutes. No noticeable changes in strength or structure have been observed since the first measurements of "Pioneer -10" in mid-1970 [41].

Size and Shape of Jupiter - Jupiter's internal magnetic field interferes with the solar wind's stream of ionized particles flowing out of the solar upper atmosphere, preventing streams of ions from reaching Jupiter's atmosphere, deflecting them away from the planet and creating a kind of cavity in the solar wind called the magnetosphere, which is made of plasma, different from solar wind plasma. The Jovian magnetosphere is so large that if you place the Sun in it, even with its visible corona, there will still be enough space there [38]. The region between the magnetopause and the bow shock is called the magnetic transition layer, or magneto sheath (see [41]). Schematic representation of the magnetosphere, where the plasma sphere (Figure 2), (7) faces the torus of plasma and the magneto layer.

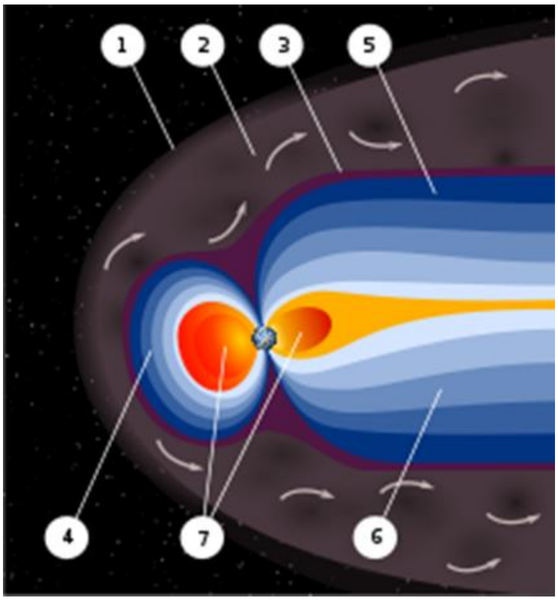

Figure 2. *Schematic representation of the magnetosphere, where the plasma is a sphere.*

Beyond the night side of the planet, the solar wind extends the lines of Jupiter’s magnetic field into a long, elongated tail of the magnetosphere, which sometimes extends even beyond the orbit of Saturn. In its structure, the tail of the Jovian magnetosphere is reminiscent of the Earth’s. It consists of two “petals” (areas marked in blue in the diagram). The magnetic field in the southern petal is directed towards Jupiter, and in the northern petal - away from it. The lobes are separated by a thin layer of plasma called the tail current sheet (the elongated orange zone in the diagram). Like the Earth's, the Jovian magneto tail is a channel through which solar plasma enters the inner regions of the magnetosphere, where it heats up and forms radiation belts at a distance of less than 10 R_J from Jupiter [38]. The shape of Jupiter's magnetosphere described above is maintained by 1) a neutral current sheet (also known as a magneto tail current) that flows in the direction of Jupiter's rotation through the tail plasma layer (see Figure 2) plasma flows within the tail flowing against Jupiter's rotation at the outer boundary magneto tail, and (see Figure 2) magnetopause currents (or Chapman-Ferrari currents), which flow against the rotation of the planet on the dayside of the magnetopause. These currents create a magnetic field that cancels out (compensates) Jupiter's internal field outside the magnetosphere. They also actively interact with the solar wind [41]. Traditionally, Jupiter's magnetosphere [46] is divided into three parts: the inner, middle, and outer magnetospheres. The inner one lies at a distance of up to 10 R_J from the center of the planet. The magnetic field inside it is predominantly a dipole because the contribution from currents passing through the equatorial plasma layer is very insignificant. In the middle (between 10 and 40 R_J) and outer (hereinafter 40 R_J) magnetospheres, the magnetic field deviates from the dipole structure and is seriously perturbed by the influence of the plasma layer (see [37, 38]).

2.3. The Role of Jupiter's Satellite Io, Source as a Hot Plasma Source for Jupiter

Although in general the magnetosphere of Jupiter resembles the Earth's in shape, close to the planet their structures are very different [39]. Io, a volcanically active satellite of Jupiter, is a powerful source of plasma and every second replenishes the magnetosphere of Jupiter with ~1000 kg of new matter. Strong volcanic eruptions on Io lift into the open space is sulfur dioxide, most of which dissociates into atoms and is ionized by solar ultraviolet radiation.

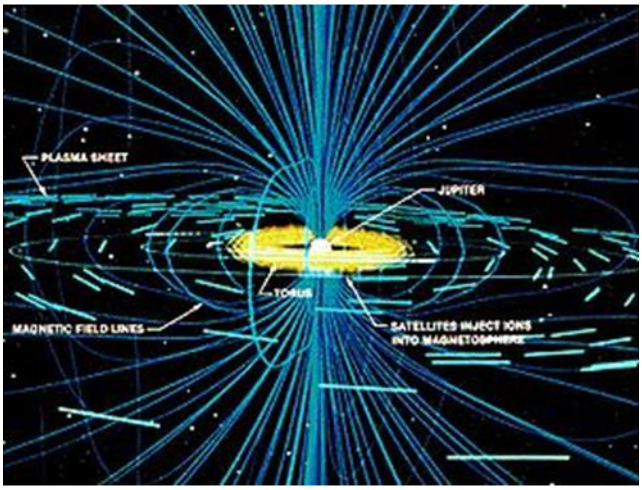

Figure 3. *Interaction of Io with the magnetosphere of Jupiter. Io's plasma torus highlighted in yellow.*

As a result, sulfur and oxygen ions are formed: S^+, O^+, S2+, and O_2^+ ions leave the satellite's atmosphere, forming Io's plasma torus, a massive and relatively cold ring of plasma surrounding Jupiter along the satellite's orbit [39]. The temperature of the plasma inside the torus reaches 10-100 eV (100,0001,000,000 K), which is much lower than the energy of particles in the radiation belts, which is 10 KeV (100 million K). The plasma inside the torus is driven by the magnetic field of Jupiter "frozen" into it into rotation with the same period as Jupiter itself [44] (such synchronous rotation is called corotation). The torus of Io has a significant impact on the dynamics of the entire magnetosphere of Jupiter. As a result of several processes, among which the main role is played by diffusion and exchange instability, the plasma slowly leaves the vicinity of the planet [40]. As the plasma moves away from Jupiter, the radial currents flowing through it gradually increase their speed, maintaining coronation [38]. These radial currents also serve as a source of the azimuthal component of the magnetic field, which, as a result, bends backward relative to the direction of rotation. The concentration of particles in the plasma decreases from 2000 sm^{-3} in the torus of Io to about 0.2 sm^{-3} at a distance of 35 R_J [42]. In the middle magnetosphere, at a distance of more than 20 R_J from Jupiter, corotation gradually stops, and the plasma rotates more slowly than the planet [40]. Ultimately, at a distance of more than 40 R_J (in the outer magnetosphere), the plasma finally leaves the magnetic field and goes into interplanetary space through the tail of the magnetosphere. Moving outward, the cold, dense plasma exchanges places with the hot, rarefied plasma (with a temperature of 20 KeV (200 million K) or higher) moving from the outer magnetosphere. This plasma, approaching Jupiter and compressing, is heated adiabatically, forming radiation belts in the inner magnetosphere (see [42-44]).

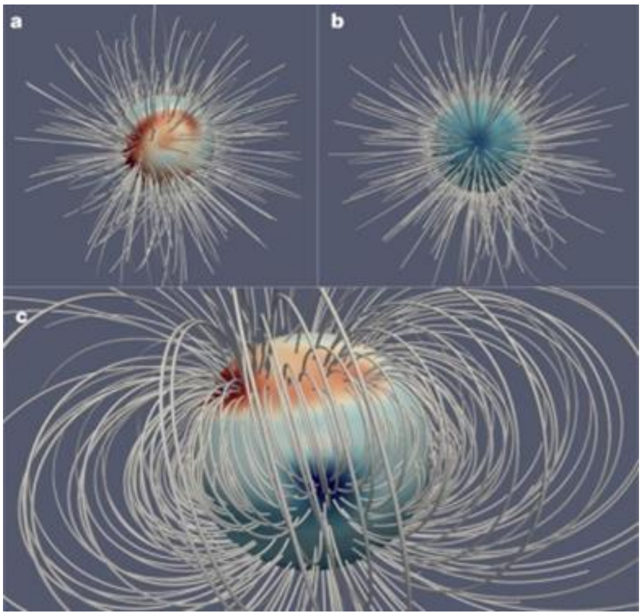

Figure 4. *Scheme of Jupiter's magnetic field lines. A view of the north pole; B view of the south pole; C view of the equator.*

The non-dipole nature of the magnetic field in the northern hemisphere and the dipole in the southern hemisphere are shown. The outlined sphere represents the proposed boundary of the metallic hydrogen core; whose radius is equal to 0.85 of the radius of Jupiter. (Nature 2018; Moore et al).

2.4. The Magnetic Disk Acts as a Balancing Mechanical Equilibrium to Retain Hot Plasma

Unlike the Earth's magnetic field, which is approximately teardrop-shaped, Jupiter's field is more flattened, more like a disk, and periodically oscillates about its axis. The main reason for this disk-shaped configuration is the centrifugal forces caused by the coronation of the plasma and magnetic field, as well as the thermal pressure of the hot plasma. Both phenomena lead to the stretching of magnetic field lines, forming a flattened, pancake-shaped structure known as a “magnetic disk” at a distance of more than 20 R_J from the planet. The magnetic field lines are directed from Jupiter above this layer and to Jupiter below it. Plasma coming from Io significantly increases the size of Jupiter's magnetosphere since the magnetic disk creates additional internal pressure that balances the pressure of the solar wind. The distance from the planet to the magnetopause at the “subsolar point, » equal on average to 75 R_J, would decrease to 42 R_J in the absence of Io [43]. It should be noted that most of Jupiter's magnetic field, like Earth's, is generated by an internal dynamo maintained by the circulation of electrically conductive fluid in the outer core. But while the Earth's core is made of molten iron and nickel, Jupiter's core is made of metallic hydrogen. Maxwell's fourth equation applies and shows that such loops of electric current generate a magnetic field. Changes in this magnetic field, according to Faraday's law of induction (Maxwell's third equation), generate an electric field. These electric and magnetic fields jointly act on particles (electric on any particles, magnetic only on moving ones) by the Lorentz force, accelerating their movement according to Newton’s second law, and a positive feedback loop arises. All these relationships can be described using a partial differential equation, which forms the basis of the theory of magnetic dynamo (also similar to that noted by E. Parker [46]; the reconnection process is always restored), which explains the existence of magnetospheres near the Earth and other planets of the solar system, as well as near the Sun itself (only in the Sun is the role of a conducting liquid played by ionized gas).

3. Important Explanations: Derivation of the Cold Plasma Modeling Equations

Derivation of the Cold Plasma Modeling Equation Whose Source is Jupiter's Satellite Io

Based on the considerations noted above in sections 1.1, 1.2, 2.1, 2.2, 2.3, 2.4 and 2.5, as well as standard methods for deriving models of plasma, electromagnetic and Lorentz force, applications of Newton’s law of motion, Maxwell’s law, and dielectric tensor cold plasma, we obtain a mathematical model, plasma equations, which are justified by the description of non-classical equations of mathematical physics. Accordance above physical process of Jupiter’s satellite Io let’s consider derivation of equation model cold plasma, which move to the Jupiter’s magnetosphere.

Consider a single particle of mass m, having charge $q = Z\delta e$, where Z is a positive integer, δ equals 1 or −1, and e is the charge on an electron. Let the particle be subjected only to the Lorentz force:

$$\vec{F}_{Lorentz} = \vec{F}_{Elec} + \vec{F}_{Mag}\ ,\ \vec{F} = q(\vec{E} + \vec{U} \times \vec{B}) \tag{1}$$

$$\vec{B} = B_0\, \vec{k} \tag{2}$$

Equation (1) implies that the applied magnetics field is longitudinal: its only nonzero component is directed along the positive z-axis. (In fact, there is little harm in assuming, somewhat more generally, that $\vec{B} = B_0\, \vec{k} + \vec{B}_1(x, y, z) e^{i(\vec{k} r - \omega t)}$, where $\left|\vec{B}_1\right| \le \left|B_0\right|$ As known that the equation of motion for particle is given by Newton’s Law of Motion in the following formula

$$m\frac{d\vec{U}}{dt} = \vec{F} \tag{3}$$

Hence, from the equation (1) we get

$$\vec{U}(x, y, z, t) = \vec{U}_1(x, y, z, t) e^{i(\vec{k} r - \omega t)},\quad \text{or}\quad \frac{d\vec{U}}{dt} = -i\omega\vec{U} \tag{4}$$

Account into (3) in (4) we have $-im\omega\vec{U} = q(\vec{E} + \vec{U} \times \vec{B})$ where $\vec{E} = \vec{E}_1(x, y, z) e^{i(\vec{k} r - \omega t)}$, $\vec{E}_1(x, y, z) = (E_1)_1\, \vec{i} + (E_1)_2\, \vec{j} + (E_1)_3\, \vec{k}$, and the components $(E_1)_1, (E_1)_2, (E_1)_3$ are constants in with type shown in $\vec{U}(x, y, z, t)$.

$f_{cyclotron} = f_c = \left|\frac{qB_0}{m}\right|$ where f is the (linear) frequency, q is the charge of the particle, B is the magnitude of the magnetic field that is perpendicular to the plane in which the particle is travelling, and m is the particle mass and with the cyclotron frequency equation to yield: $U = \frac{qBr}{m}$. Hence, the kinetic energy for particles with speed U can be written by

$E_{kinetic} = \frac{mU^2}{2} = \frac{qB^2r^2}{2m}$ and we can find $\vec{U} = (u_1?u_2, u_3)$ by the components:

$$\begin{cases} u_1 = \frac{iq}{m(\omega^2 - f_c^2)}(\omega(E_1)_1 + i\delta f_c(E_1)_2), \\ u_2 = \frac{iq}{m(\omega^2 - f_c^2)}(\omega(E_1)_2 - i\delta f_c(E_1)_1), \\ u_3 = \frac{iq}{m\omega}(E_1)_3. \end{cases} \tag{5}$$

Although the above relations were derived for an individual particle, they also hold, in our simplified linear model, for each species of particle in a plasma consisting of electrons and N − 1 species of ions. In particular, the plasma current can be written as the sum

$$\vec{J} = \sum_{\nu=1}^{N} n_\nu q_\nu U_\theta \tag{6}$$

where n_ν is the density of particles having charge magnitude $|q_\nu| = Z_\nu e_\theta$. In the sequel we will only consider the aggregate of particles, in which equation of (1)-(6) pertain with the quantities $U, m, q, Z, \delta,$ f_c indexed by ν where $\nu = 1, ..., N$. Introduce the electric displacement vector in the following form $\vec{D} = \varepsilon_0 E + \frac{i}{\omega}\vec{J}$ where ε_0 is the permittivity of free space and we can convenient to express in the form

$$\begin{cases} \vec{D} = \varepsilon_0 \vec{K}\vec{E}, \quad \vec{D} = (D), \vec{K} = (K_{ij}), \vec{E} = (E_j), \\ D_i = \varepsilon_0 \sum_{j=1}^{3} K_{ij} E_j, \end{cases} \tag{7}$$

The $\vec{K} = (K_{ij})$, is said to be dielectric tensor and also called the cold plasma conductivity tensor. The tonsorial nature of this quantity reflects the anisotropy of the plasma due to the presence of an applied magnetics field. Equations (6), (7)

$$\text{imply that } K = (K_{ij}) = \begin{pmatrix} K_{11}K_{12}K_{13} \\ K_{21}K_{22}K_{23} \\ K_{31}K_{32}K_{33} \end{pmatrix},$$

where, $K_{11} = s$, $K_{12} = -id$, $K_{13} = 0$, $K_{21} = id$, $K_{22} = s$, $K_{23} = 0$, $K_{31} = 0, K_{32} = 0, K_{33} = p.$

In this tensor, they are elements s, d, p can be defined from the plasma frequency, permittivities R or L of a right- or left-circularly polarized wave travelling in the direction $\vec{k}$. The plasma frequency which for particles of the species is given by formula $\Pi = q_\nu \sqrt{\frac{n_\nu}{\varepsilon_0 m_\nu}}$, or $\Pi^2 = \frac{q_\nu^2 n_\nu}{\varepsilon_0 m_\nu}$. But permittivity R or L are given by

$$R_{permitt} = 1 - \sum_{\nu=1}^{N} \frac{\Pi^2}{\omega^2}\left(\frac{\omega^2}{\omega^2 + \delta_\nu (f_c)_\nu}\right), \quad L_{permitt} = 1 - \sum_{\nu=1}^{N} \frac{\Pi^2}{\omega^2}\left(\frac{\omega^2}{\omega^2 - \delta_\nu (f_c)_\nu}\right). \tag{8}$$

$$\text{Hence,} \quad s = 2 - \sum_{\nu=1}^{N} \frac{\Pi^2\omega^2}{\omega^4 - \delta_\nu^2 (f_c)_\nu^2}, \quad d = \sum_{\nu=1}^{N} \frac{\Pi^2\delta_\nu}{\omega^4 - \delta_\nu^2 (f_c)_\nu^2} \quad p = 1 - \sum_{\nu=1}^{N} \frac{\Pi^2}{\omega^2} \tag{9}$$

Note that, the mass of an electron is considerably smaller than the mass of any ion; so the squared ion cyclotron frequencies (references to "the plasma frequency" in textbooks invariably mean the electron plasma frequency) obtained from combining fractions in $R_{permitt}$ and $L_{permitt}$ can be neglected and therefore, the ion plasma frequencies can be neglected in the definition of $p = 1 - \sum_{\nu=1}^{N} \frac{\Pi^2}{\omega^2}$ (usually, it is standard notation). It is easily seen that Π -corresponds to the typical electrostatic oscillation frequency of a given species in response to a small charge separation. Now, accordance to the Maxwell's we can write the following

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{\nabla} \times \vec{B} = \mu_0(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}) \tag{10}$$

where, the parameter μ_0 denoted by the permeability's of free space. As known that from the equations of (4) the $\vec{E}$ and $\vec{B}$ whenever implies plane wave, therefore the system equations of (10) can be expressed by

$$\begin{cases} \vec{k} \times \vec{E} = \omega \vec{B} \\ \vec{k} \times \vec{B} = -i\mu_0 \vec{j} - \omega\mu_0\varepsilon_0 \vec{E} \end{cases} \tag{11}$$

Hence, we can write $\vec{k}\times\vec{B}=\varepsilon_0\omega\mu_0\vec{K}\vec{E}$. Hence, account into system of (11) and using elementary identity $\mu_0\varepsilon_0=\frac{1}{c^2}$ (where c- is speed of light in vacuum) and from the equalities $\vec{k}\times(\vec{k}\times\vec{E})+\left(\frac{\omega}{c}\right)^2\varepsilon_0\vec{K}\vec{E}=0,\quad \vec{n}=\frac{c}{\omega}\vec{k}$ we may obtain the following matrix:

$$\begin{bmatrix} s-n^2\cos^2(\phi) & \text{-id} & n^2\cos(\phi)\sin(\phi) \\ \text{id} & \text{s-n}^2 & 0 \\ n^2\cos(\phi)\sin(\phi) & 0 & p^2\text{-}n^2\sin^2(\phi) \end{bmatrix}\begin{bmatrix} E_{1,1} \\ E_{1,2} \\ E_{1,3} \end{bmatrix}=0. \tag{12}$$

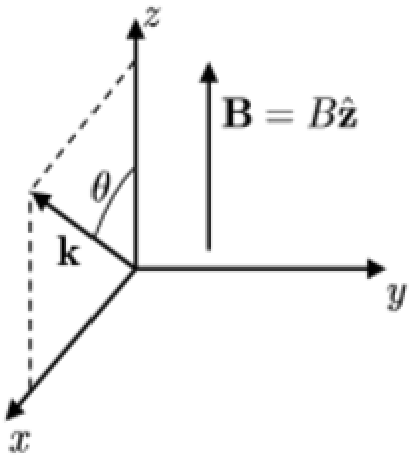

Figure 5. *Cold plasma waves.*

As known that from the theory linear algebraic homogeneous system equation, in order existence nontrivial solution must be corresponding determinant equal to zero. From determinant equation (12) we find n^2:

$$n^2=\frac{[(s^2-d^2)\sin^2(\phi)+ps(1+\cos^2(\phi)]\pm\sqrt{[(s^2-d^2)\sin^2(\phi)+ps(1+\cos^2(\phi)]^2-4AC}}{2[s(\sin^2(\phi)+p\cos^2(\phi)]} \tag{13}$$

Where $A=[s(\sin^2(\phi)+p\cos^2(\phi)]$, $C=p(s^2-d^2)$. In equality (13) both sides squaring and after account into $R_{permitt}=R$ and $L_{permitt}=L$ expression and value of A and C, we have $\tan\phi=-\frac{p(n^2-R)(n^2-L)}{(sn^2-RL)(n^2-p)}$. These equations yield criteria for cutoff when $n=0$, or rezones when $n\to\infty$. Note that. Physically, cutoffs and resonances correspond to a change in the behavior of the wave from possible propagation to evanescence. Mathematically, we will identify certain resonances with a change in the type of governing field equation from hyperbolic (involving wave propagation) to elliptic (involving outflow). These transitions, under certain conditions, can be accompanied by the reflection and/or absorption of the wave. Sufficient conditions for cutoff are $p=0,R=0$ or $L=0,C=0$. But for resonances sufficient conditions $A=[s(\sin^2(\phi)+p\cos^2(\phi)]=0$, it means that $\tan\phi=-\frac{p}{s}$.. Physically, means that for propagation parallel to the magnetic fled must be satisfying satisfy condition of $\phi=\frac{\pi}{2}$. We will be particularly, interested in the hybrid resonances at $\phi=\frac{\pi}{2}$, which occur at frequencies for which s = 0. Remember that, the electric field is said to be electrostatic if it approximately satisfies $E=-\nabla\Phi$, where Φ is a scalar potential and this equation is satisfied locally by all-time independent electric fields and in an ordinary dielectric, the converse is also true. However in cold plasma there also exist time dependent solutions of $E=-\nabla\Phi$. Cold plasma has been characterized as a linear dielectric through which electromagnetic waves propagate. Thus these waves include, in distinction to ordinary dielectrics, the special case of propagating electrostatic waves. Additionally, adding from Gauss law's for electricity, we get

$$\begin{cases} \vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}, \\ \vec{\nabla}\times\vec{B} = \mu_0(\vec{j}+\varepsilon_0\dfrac{\partial\vec{E}}{\partial t} \\ divD = 0, \\ E = -\nabla\Phi, \Phi(x,y,z,t) = \Phi_1(x,y,z,t)e^{(\vec{k}\,r-i\omega t)}, \end{cases} \quad (14)$$

Hence we get $\vec{\nabla}\times\vec{E} = 0$. Let's consider the following cases: $\vec{\nabla}\vec{B} = 0$.

(i). If we allow a plane-layered inhomogeneous medium (parameterized by x), the electrostatic potential has the form $\Phi(x,y,z) = u(x)e^{i(k_2y+k_3z)}$, where $\vec{k} = (k_1,k_2,k_3)$ is propagation vector of wave. Substitution of this form for the electric potential into equation $divD = 0$ and using $D_i = \varepsilon_0\sum_{j=1}^{3}K_{ij}E_j$ (see [21]) we get

$$K_{11}u_{xx} + ((K_{11})_x + i\sigma_0)u_x = 0, \quad (15)$$

where, $\sigma_0 = k_3(K_{13}+K_{31}) + k_2(K_{12}+K_{21})$, and zero order $u(x)$ is neglected. The equation of (14) has a power –series solution with except when $K_{11} = 0$. Explicit solutions of the model equation (15) under various physical assumptions are given ([5, 6, 20, 21, 22].). It is easy to believe that inhomogeneities may develop in a plasma. For example, if the temperature is not exactly zero, the difference in velocity between electrons and ions can be expected to destabilize an initially homogeneous distribution. However, it is difficult to imagine a force that will restrict these inhomogeneities to a 1-parameter foliation, which would be necessary in order to arrive at equation (15). Formally, an electromagnetic potential leading to equation of (15) could be induced by applying a driving potential to the metallic plates of a condenser. But in practice, this plasma geometry has little application either in the laboratory or in nature.

(ii). Suppose instead that the medium is a cold, anisotropic plasma with a two-dimensional inhomogeneity parameterized by two variables, x and z (see [20]). Then the field potential has the form $\Phi(x,y,z) = u(x,z)e^{ik_2y}$. As known the electric field $\vec{E} = \nabla\vec{\Phi} = (E_1,E_2,E_3)$, where

$$E_1 = u(x,z)e^{ik_2y}, \quad E_2 = u_x(x,z)ik_2e^{ik_2y}, \quad E_3 = u_z(x,z)k_2e^{ik_2y},$$

for electric displacement vector $\vec{D} = (D_1,D_2,D_3)$, taken Maxwell's equation with $div\,\vec{D} = 0$, and hence we get

$$0 = \nabla\vec{D} = (D_1)_x + (D_2)_y + (D_3)_z \quad (16)$$

We continue to neglect those terms which do not contain derivatives of $u(x,z)$ as $u(x,z)$ is assumed to oscillate rapidly. Because neither $u(x,z)$ nor K_{ij} have any dependence on y, the problem is two-dimensional case taken. Using (14) and the $D_i = \varepsilon_0\sum_{j=1}^{3}K_{ij}E_j$ when $\varepsilon_0 = 1$ and collecting terms we find that [20]:

$$K_{11}u_{xx}(x,z) + 2\sigma u_{xz}(x,z) + K_{33}u_{zz} + a(x,z)u_x + b(x,z)u_z = 0, \quad (17)$$

$2\sigma = K_{13} + K_{31}$, $a(x,z) = (K_{11})_x + ik_2(K_{12}+K_{21}) + (K_{31})_z$, $b(x,z) = (K_{13})_x + ik_2(K_{23}+K_{32}) + (K_{33})_z$. If for matrix K to make under our assumptions on $\vec{B} = 0$ imply that $\sigma = 0$. Therefore, the equation (15) can be written in the following form

$$K_{11}u_{xx}(x,z) + K_{33}u_{zz} + a(x,z)u_x + b(x,z)u_z = 0, \quad (18)$$

If take in simple notation as $K_{11} = K_1(x), K_{33} = K_2(z)$ then

$$K_1(x)u_{xx}(x,z) + K_2(z)u_{zz} + a(x,z)u_x + b(x,z)u_z = 0, \quad (19)$$

The equation of (18) is the particular case of equation M. A. Nurmammadov ([13, 33-35]).

$$K_1(x)u_{xx}(x,z) + K_2(z)u_{zz} + a(x,z)u_x + b(x,z)u_z + c(x,z) = f(x,z), \quad (20)$$

When $xK_1(x) > 0$, for $x \neq 0$, $zK_1(z) < 0$, for $z \neq 0$. The equation is called mixed elliptic-hyperbolic type of M. V. Keldysh in two dimensional problems. Two-dimensional inhomogeneities of the kind represented by equations of (18) and (19), (20) can be expected to arise in toroidal fields, such as those created in tokamaks. Finally, frankly speaking, the cold plasma and toroidal –poloidal plasma which in tokamaks describing mathematically models corresponding to elliptical-hyperbolic equation of mixed type, both Keldysh and Tricomi or Keldysh-Trikomi types. In last time these equations is called equations of nonclassical mathematical physics (see above of definition as it noted in section "Introduction'). Now, let's consider for equation (20) when will be elliptical or hyperbolic? For cold plasma in anisotropy case, the equation (20) is of either elliptical or hyperbolic type, depending on whether the sign of the product $K_{11}K_{33}$ or K_1K_2, i. e. from sign

$$K_1K_2 = (1-\sum_{\nu=1}^{N}\frac{\Pi_\nu^2}{\omega^2-(f_c)_\nu^2})(1-\sum_{\nu=1}^{N}\frac{\Pi_\nu^2}{\omega^2}). \qquad (21)$$

is, respectively, positive or negative. In this case, the sign of K_1 changes at the cyclotron resonances $\omega^2 = (f_c)_\nu^2$. The cold plasma model breaks down at these resonances, as three terms of the dielectric tensor become infinite. Also, the sign of K_1 as changes at the hybrid resonances, at which satisfy condition

$$1 = \sum_{\nu=1}^{N}\frac{\Pi_\nu^2}{\omega^2-(f_c)_\nu^2} \qquad (22)$$

(These resonances, which have both a low-frequency and a high-frequency solution, are hybrid in that they involve both plasma and cyclotron frequencies.) In particular, the sign changes at the lower hybrid resonance (standard form)

$$1-\frac{\Pi_\nu^2}{\omega^2-(f_c)_\nu^2} = \frac{\Pi_\nu^2}{\omega^2} \qquad (23)$$

The sign of K_{33} changes on the surface

$$1 = \sum_{\nu=1}^{N}\frac{\Pi_\nu^2}{\omega^2} \qquad (24)$$

where as before, the subscript e denotes electron frequency, and the subscript i denotes ion frequency. At the hybrid resonance frequencies, the cold plasma model retains its validity. Thus in evaluating (21) and in the sequel we will take K_2 to be strictly positive. The sonic condition of $K\sin^2(\phi)+\eta\cos^2(\phi)=0$, include that the x-axis is directed along the inward normal to the sonic line, relative to the hyperbolic region of equation (18) (or (19)). Hence, $K_{11}=0, \sigma=0$ at the origin and taking both x and z to be small, one can write $K_{11} = \frac{x}{a}+\frac{z^2}{b}$, $K_{33} = -\eta_0$, for constant $\eta_0 > 0$. If we take scale x and z in the form $x \to x_1 = \frac{x}{a}, z \to z_1 = \frac{z}{a\sqrt{\eta_0}}$ then we have

$$-(x_1+Cz_1^2)u_{x_1x_1}(x_1,z_1)+u_{z_1z_1}-u_{x_1}=0, \qquad (25)$$

where C- is constant, for example, if instead of $-(x_1+Cz_1^2$, take $z_1^2-x_1$, then cold plasma equation can be written as $(x_1+z_1^2)u_{x_1x_1}-u_{z_1z_1}+u_{x_1}=0$, where the sonic line is parabola line $x_1 = z_1^2$. Physical reasoning suggests that the closed Dirichlet problem, in which data are prescribed along the entire boundary of the domain, should be well-posed for the cold plasma model convenience for on a typical domain [24] it will be rather.

$$(x-z^2)u_{xx}(x,z)+u_{zz}+\frac{1}{2}u_x=0. \qquad (26)$$

Continuing to adopt the special hypotheses and special notation, we can continue to review the analysis in [23] of geometry-preserving plane waves in an axisymmetric plasma, which is equation of mixed elliptic-hyperbolic type. In the work (see [18, 20]), the features of the field in a cold anisotropic plasma with two-dimensional inhomogeneity are considered in connection with the problem of converting electromagnetic waves into plasma waves. The nature of the field singularities is determined by the type of singular point of the characteristic (a pass leads to a singularity on a line, a node leads to a singularity at a point). The mutual linear transformation of electromagnetic and plasma waves in an inhomogeneous plasma is of considerable interest and has been studied by many; see, for example, [14-21]. As is known, the linear transformation of electromagnetic waves into plasma waves in a flat layer is closely related to the field characteristics of cold plasma. It is the presence of such features that is a necessary condition for the transformation, and the energy carried away by the plasma wave is equal to the energy absorbed in the cold plasma. This circumstance makes it essentially unnecessary to solve the transformation problem if the simpler problem for cold plasma is solved. This article is devoted to the study of field features in the case when the parameters of the environment depend on two spatial coordinates. The two-dimensional inhomogeneity model covers a large number of plasma configurations, including a Tokamak-type toroidal system. The considerations expressed above are, of course, purely qualitative in nature. In this section, we give a more rigorous analysis, which, in particular, will allow us to establish the type of field features that arise in the indicated “special” lines and special points in the magnetosphere of Jupiter.

4. Solution Near Singular Points of the Characteristics of the Cold Plasma Equation

The considerations expressed above are, of course, purely qualitative in nature. In this section, we present a more rigorous analysis, which, in particular, will allow us to establish the type of field singularities that arise in the indicated “special” lines and special points. Mixed-type equations, such as equation (25), describe model anisotropic cold plasma flowing from the satellite Io of the Jupiter magnetosphere. In this sense, as a study from a mathematical point of view, the presented model, described by the use of non-classical equations of mathematical physics, which included the Tricomi and Keldysh equations, was studied for the first time. In order to have motivation for non-classical models, we first illustrate a special solution for the Tricomi, Keldysh, and Tricomi-Keldysh type model equations.

$$zu_{xx}(x,z)+u_{zz}=0,$$
$$K(z)u_{xx}(x,z)+u_{zz}=0, K(z)=z,z^2,z^m \quad (27)$$

Keldysh type equation deriving from equation of Tricomi (27) in the form

$$K^{-1}(z)u_{zz}(x,z)+u_{xx}(x,z)=0. \quad K(z)=z,z^2,z^m$$

For Tricomi (27) equation some exact solutions formulas exist ([3]), and solutions to some particular boundary values problems are known ([47, 48]). Let's consider more general form for equation (25) with coefficient $(x+\alpha z^2)$:

$$(x+\alpha z^2)u_{xx}(x,z)-u_{zz}+\frac{1}{2}u_x=0. \quad (28)$$

If we seek the solution of (28) in the form

$$\phi=u^{\beta}\xi^{p}\upsilon(\zeta),\ u=x+\frac{\beta}{2}z^2,\ \xi=u^{2\beta}z,\ \zeta=\frac{4u}{(1+4\beta)z^2}, \quad (29)$$

where p is an arbitrary constant, then we obtain for the function $\upsilon(\zeta)$ the hypergeometric equation

$$\zeta(1-\zeta)\frac{\partial^2\upsilon}{\partial\zeta^2}+[1+\beta(2p+1)-(\frac{3}{2}-p)\zeta]\frac{\partial\upsilon}{\partial\zeta}-\frac{p(p-1)}{4}\upsilon=0. \quad (30)$$

Almost works which is concerning plasma and its equation being to the form (30) and it can be solved Bessel solutions. But in the work author M. A. Nurmammadov [13] considered non-classical equations of mathematical physics, and founded justification classes a new ordinary equations non-classical type, which are has application astrophysics arise Kelvin-Helmholtz instabilities. In this case NODE (non-classical ordinary differential equation) has degenerating lines and singular points. For example in equation (30) has degenerating points $\zeta=0,\zeta=1$, or singularities.

$K(x)=x(L-x), x=0, x=L, b(x)\geq 0 (or b(x)>0)$, or the number $b(x)=b>0$

$$x(1-x)u_{xx}(x)+b(x)u_x(x)+c(x)u=f(x), \quad (31)$$

In this situation, The coefficients of (30) corresponding for (28) in the following:

$$K(\zeta)=\zeta(1-\zeta), b(\zeta)=[1+\beta(2p+1)-(\frac{3}{2}-p)\zeta],$$
$$c(\zeta)=-\frac{p(p-1)}{4} \quad (32)$$

Accordance, equation of (33) using the solution can be written in the following integrals formulas,

$\upsilon(\zeta)=C_1(\exp(P(\zeta))+C_2(\exp(P_1(\zeta))$, where $P(\zeta)=\int_{\zeta}^{1}\frac{c(t)}{b(t)}dt,$

$P_1(\zeta)=\int_{\zeta}^{1}\frac{c(t)}{t(1-t)}dt,$ and C_1, C_2 are arbitrary constants which choosing from the initial conditions.

$$P(\zeta)=-\int_{\zeta}^{1}\frac{p(p-1)}{4[1+\beta(2p+1)-(\frac{3}{2}-p)t]}dt, \qquad P_1(\zeta)=-\int_{\zeta}^{1}\frac{p(p-1)}{4t(1-t)}dt$$

Hence we get

$$\upsilon(\zeta)=C_1(\exp[-(\frac{p(p-1)}{6})\ln\left|1+\beta(2p+1)-(\frac{3}{2}-p)t\right|\Big|_{t=\zeta}^{t=1}]+C_2(\exp(-\frac{p(p-1)}{4}\lim_{A\to 1}\ln t(1-t)\Big|_{t=\zeta}^{t=A})$$

Analogically for equation $(x-z^2)u_{xx}(x,z)-u_{zz}+\frac{1}{2}u_x=0,$ can be taken $\phi=u^{\beta}\xi^{p}\upsilon(\zeta),\ u=x-\frac{\beta}{2}z^2,\ \xi=u^{2\beta}z,$ $\zeta=\frac{4u}{(1+4\beta)z^2}$, then $\alpha=-1,\beta=-2$, and $p<0$ arbitrary number is applicable for this equation. As it shown in above the coefficients of (30) can be written in the following form

$$\zeta(1-\zeta)\frac{\partial^2\upsilon}{\partial\zeta^2}+[-(1+4p)-(\frac{3}{2}-p)\zeta]\frac{\partial\upsilon}{\partial\zeta}-\frac{p(p-1)}{4}\upsilon=0. \quad (33)$$

$$K(\zeta)=\zeta(1-\zeta), b(\zeta)=[-(4p+1)-(\frac{3}{2}-p)\zeta],\ c(\zeta)=-\frac{p(p-1)}{4} \quad (34)$$

Accordance, equation of (33) using this case solution can be written in the following integrals formulas,

$\upsilon(\zeta) = C_1(\exp(P(\zeta)) + C_2(\exp(P_1(\zeta))$, where $P(\zeta) = \int_{\zeta}^{1} \frac{c(t)}{b(t)} dt$, $P_1(\zeta) = \int_{\zeta}^{1} \frac{c(t)}{t(1-t)} dt$, and C_1, C_2 are arbitrary constants which choosing from the initial conditions.

$$P(\zeta) = -\int_{\zeta}^{1} \frac{p(p-1)}{4[-(4p+1)-(\frac{3}{2}-p)t]} dt, \quad P_1(\zeta) = -\int_{\zeta}^{1} \frac{p(p-1)}{4t(1-t)} dt \tag{35}$$

Finally, we get

$$\begin{aligned}\phi(x,z) &= (x-z^2)^{-6}[\frac{(x-z^2)}{-7z^2}]^p [C_1(\exp[-(\frac{p(p-1)}{6}) \ln\left|[-(4p+1)-(\frac{3}{2}-p)t\right| \Big|_{t=\zeta}^{t=1}] + \\ &C_2(\exp(-\frac{p(p-1)}{4} \lim_{A\to 1} \ln t(1-t)\Big|_{t=\zeta}^{t=A}).,\ \zeta = -\frac{4}{7}(\frac{x}{z^2}-1)\end{aligned} \tag{36}$$

or in our terminology we have the following solution

$$\begin{aligned}u(x,z) &= (x-z^2)^{-6}[\frac{(x-z^2)}{-7z^2}]^p [C_1(\exp[-(\frac{p(p-1)}{6}) \ln\left|[-(4p+1)-(\frac{3}{2}-p)t\right| \Big|_{t=\zeta}^{t=1}] + \\ &C_2(\exp(-\frac{p(p-1)}{4} \lim_{A\to 1} \ln t(1-t)\Big|_{t=\zeta}^{t=A}).,\ \zeta = -\frac{4}{7}(\frac{x}{z^2}-1)\end{aligned}$$

In case of $\alpha < -1$ in equation of (28) does not a single characteristics passes through the point $x = 0, z = 0$. but when $-1 < \alpha < 0$ there are infinity numbers characteristics passes through the point $x = 0, z = 0$. Additionally, all the characteristics lying between the parabolas $x = -z^2$ and $x = \alpha z^2$, when $\alpha > 0$ one characteristics from the each family passes point of $x = 0, z = 0$. When $z = 0$ singularity case equation is $xu_{xx}(x,z) - u_{zz} + \frac{1}{2}u_x = 0$ and there exists, solutions having a strong singularity on the line $x = z^2$ (this line is the second characteristic parabola). A similar situation occurs also in the case of plane geometry, when $\beta = \alpha = 0$, the equation (28) describes the case of plane geometry with the concentration gradient perpendicular to the magnetic field. Finally, the solutions should have a singularity along the entire line representing the corresponding branches of the characteristics when z < 0 and z > 0.

5. Equilibrium Between Magnetic and Centrifugal Forces, Pressure Gradient, in the Presence of Plasma Rotations

Note that Caudal G. [49] studied a stationary axisymmetric magnetosphere (with the establishment of axial symmetry relative to the axis of rotation of the magnetic axis, where denotes the azimuthal direction in spherical or cylindrical coordinates). The displacement of the centrifugal equator (toward the magnetic equator) is expected to be small up to distances of 30 RJ and is therefore not included in our study. Again, we ignore any temporal or spatial variations, e.g., by density, since they usually have a small size (< 1) or are very short (several minutes). The model also assumes that there is enough time to connect the magneto disk to the planet through currents. Assuming a balance of forces in both the (cylindrical) radial and meridional directions, the equilibrium reaction between magnetic force, pressure gradient, and centrifugal force has the following equalities:

$$\vec{F}_{magnit} - \vec{\nabla} P + N_i m_i \rho \omega^2 \vec{\rho}_1 = 0, \text{ or}$$

$$\vec{J} \times \vec{B} - \vec{\nabla} P + N_i m_i \rho \omega^2 \vec{\rho}_1 = 0, \tag{37}$$

In the above expression: $\vec{J}, \vec{B}$ current density and magnetic field, respectively; P -total thermal pressure of the plasma; n_i -ion number density; m_i -average ion mass; ρ - cylindrical distance from the axis of rotation; $\vec{\rho}_1$ -cylindrical radial unit vector, angular velocity of the plasma. The contribution of the gravitational force beyond a few radii of Jupiter is minimal and can be safely ignored. In addition, following the Gaudal model [49], using the condition $\vec{\nabla} \cdot \vec{B} = 0$, we have

$$\vec{B} = \vec{\nabla} U \times \vec{\nabla} \beta_E \tag{38}$$

where the scalar functions U and β_E represents the Euler potentials (the subscript E, is used to avoid confusion with plasma β), in the general case functions in spherical coordinates. Using the rotational symmetry of the system, the Euler potentials can be simplified and rewritten as follows U:

$$U = \vec{\nabla} U \times \vec{\nabla} \beta_E, \; U = U(r,\theta), \quad \beta_E = R_{Jupiter} U \phi, \quad (39)$$

where $R_{Jupiter}$ is the radius of the planet. If we consider the meridional section of the magnetosphere, which is a constant and can be excluded from the rest of the analysis, then β_E the magnetic field in spherical coordinates is also expressed through U:

$$B_r = \frac{1}{r^2 \sin(\theta)} \frac{\partial U}{\partial \theta}, \quad B_\theta = \frac{1}{r \sin(\theta)} \frac{\partial U}{\partial r}, \quad (40)$$

where we also used the definition of azimuthal current $\vec{J}_\phi = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B}$, which is also a function Therefore, the original problem of force equilibrium then reduces to calculating only the Euler potential, which, in turn, is obtained by solving the following differential equation (in normalized units):

$$\frac{\partial^2 U}{\partial r^2} + \frac{1-\mu^2}{r^2} \frac{\partial^2 U}{\partial \theta^2} = f(r,\mu,U), \quad \text{or}$$

$$\frac{\partial^2 U}{\partial r^2} + \frac{1-\cos^2(\theta)}{r^2} \frac{\partial^2 U}{\partial \theta^2} = f(r,\mu,U), \quad (41)$$

where r - is the radial distance in spherical coordinates, μ replaces the latitude $\theta : \mu = \cos(\theta)$ and $f(r,\mu,U)$ is the source function determined by the plasma pressure and its angular velocity. This expression is physically equivalent to equation (37), in which all terms are replaced by alternative expressions involving U; basically, it includes all mechanical forces (centrifugal, pressure gradients) acting on the plasma, and we can write equation (38) in the following form:

$$\frac{\partial^2 U}{\partial r^2} + K(r,\mu) \frac{\partial^2 U}{\partial \theta^2} = f(r,\mu,U), \; K(r,\mu) = \frac{1-\mu^2}{r^2}, \quad (42)$$

$$\frac{\partial^2 U}{\partial r^2} + \frac{1-\cos^2(\theta)}{r^2} \frac{\partial^2 U}{\partial \theta^2} = f(r,\mu,U), \; K(r,\theta) = \frac{1-\cos^2(\theta)}{r^2} \quad (43)$$

then it is easy to see that equations (42) and (43) are Tricomi-Keldysh type. If $\mu = 0, \theta = \frac{\pi}{2}$, then equation (41) and (42) are Tricomi type:

$$\frac{\partial^2 U}{\partial r^2} + K_1(r) \frac{\partial^2 U}{\partial \theta^2} = f(r,0,U), \quad \frac{\partial^2 U}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 U}{\partial \theta^2} = f(r,0,U),$$

$$K_1(r) = \frac{1}{r^2}. \quad (44)$$

Keldysh type:

$$r^2 \frac{\partial^2 U}{\partial r^2} + \frac{\partial^2 U}{\partial \theta^2} = F(r,U), \quad F(r,U) = r^2 f(r,0,U), \quad (45)$$

Corollary 5.1 in orderto give motivation above obtained equations of hot –cold plasma may be conside the following equations and its special solutions:

Example 1. To find special solution of equation Tricomi: $K(z)u_{xx}(x,z) + u_{zz}(x,z) = 0., K(z) = z^2.$ Note that, $P(x,z) = z$, is a (trivial) solution. Then from the equality with $a = b = 0$, the function $u(x,z) = \int_0^x z dr - \int_0^z \int_0^t s^2 ds dt = \frac{x^2}{2} - \frac{z^4}{12}$ is a solution too.

Example2. To find special solution of equation Tricomi: $K(z)u_{xx}(x,z) + u_{zz}(x,z) = 0., K(z) = \sin^2(z)$, The function

$$u(x,z) = \int_0^x z dr - \int_0^z \int_0^t \sin^2(s) ds dt = \frac{x^2}{2} -$$

$$\frac{1}{2}[\int_0^z \int_0^t (1-\cos(2s)] ds dt = \frac{x^2}{2} - \frac{z^2}{4} - \frac{\cos(2z)}{8} + \frac{1}{4}$$

is solution. For motivation of equation (41) the function

$$u(x,z) = \int_0^x z dr - \int_0^z \int_0^t 1 - \cos^2(s) ds dt = \frac{x^2}{2} -$$

$$[\int_0^z \int_0^t (1 - \frac{1}{2}\cos(2s)] ds dt = \frac{x^2}{2} - \frac{z^2}{4} - \frac{\cos(2z)}{8} + \frac{1}{4}$$

is a solution too. If $\mu = 1, \theta = 0^\circ$, then the equation (44) and (45) are Tricomi types: $\frac{\partial^2 U}{\partial r^2} = f(r,0,U)$. Hence, we have a general solution: $\int \frac{dU}{\sqrt{C_1 + \int f(r,0,U) dr}} = C_2 \pm r,$ where C_1, C_2 are arbitrary constants. Physical means the corresponding singular point of the sonic line, cutoff, and resonance phenomena having notations -parallel propagation: the principal resonance to be these which occurs at $\theta = 0^\circ$ $\theta = \frac{\pi}{2}$. From equation (13) a resonance $n^2 \to \infty$ is evident to equation $\tan\phi = -\frac{p(n^2 - R)(n^2 - L)}{(sn^2 - RL)(n^2 - p)}$. Hence, for

$\theta \to 0^\circ$ must be satisfy the condition $S \to \infty$, since $p = 0$, is a cutoff. Since, $S = \frac{(R+L)}{2}$, then can be satisfy for either $R \to \infty$ (electron cyclotron resonance) or $L \to \infty$ (ion cyclotron resonance). But, principal solution – perpendicular propagation in such: as $\theta \to \frac{\pi}{2}$, $\frac{P}{S} \to \infty$, and since, $P \to \infty$, is a trivial solution (either $\omega \to \infty$, so, in this case $\omega_p \to \infty$, May be satisfying hybrid in Z –mode. The source function $f(r, \mu, U)$ can be used to calculate the azimuthal current $\vec{J}_\phi$, which is a key element for the stretching of field lines near the equator. By virtue of the balance between magnetic force, pressure gradient, and centrifugal force, the rotation of the plasma flow, which is a source of cold plasma, is ensured continuously by the satellite Io. In addition, the solar wind provides a trap that constantly allows the replacement of cold plasma with hot plasma, and in this region of the magnetosphere, a tail is formed, which standard MHD equations balance with the force equivalent to equations:

$$\rho(\vec{V}\vec{\nabla})\vec{V} = -\vec{\nabla} P + \frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B} \tag{46}$$

Before direct observations became available, W. Dungey [50] and T. Gold [51] observed that the magnetic fields of rotating celestial bodies impart angular velocity to the local plasma, resulting in a centrifugal force that strains the field. As a result, the field lines are stretched, or, what is the same, a current layer is formed in which the Lorentz force $\vec{J} \times \vec{B}$ in the layer balances the centrifugal force of the rotating plasma.

In the original method proposed by Caudal, the physical parameters along the equator (e.g., density, temperature) were limited by observations and in particular by Voyager 1 data [51]. In addition, a population of hot plasma (plasma with the thermal energy of the ions significantly exceeding their rotational kinetic energy) is added to the system, which contributes to the overall plasma thermal pressure P. Using data from Voyager 1 and Voyager 2, and it was reported that hot ions dominate the plasma pressure. The content of hot plasma is modeled through the product of the equatorial pressure of the hot plasma and the volume of the welding tube, which determines the hot plasma index:

$$K_{hot} = P_{hot} V_{hot} \tag{47}$$

In this case, up to a distance of ~ 8, the index is assumed to be constant with radial distance. The transfer speed itself depends on the conductivity of the ionosphere, which must also be large enough to ensure corotation of the magnetospheric plasma. Significant amounts of low-energy plasma were detected on the dayside in the radial range from 4 R_J to 5 to 16 R_J. The approximate distance at which corotation disturbances can be calculated for Jupiter from the height of the integral conductivity and external plasma flux. As an initial condition, it is assumed that the magnetic field is a dipole, described using the previous notation as:

$$U_{initial} = U_{dipol} = \frac{1 - \cos^2(\theta)}{r}, \tag{48}$$

If $\theta = 0^\circ$, then $U_{initial} = U_{dipol} = 0$, this means that beyond the critical radius, the magnetic field may be too weak to support plasma corotation and corotation violations can be calculated for Jupiter from the height of the integral conductivity and external plasma flux. If we take this violation into account from a mathematical point of view, it turns out that $C_2 = 0, C_1 = -\int f(r,0,0) = -F(r)$.

$$\int \frac{dU}{\sqrt{-F(r) + \int f(r,0,U)dr}} = \pm r, \tag{49}$$

However, the homogeneous part of the differential equation (44) has a special solution: $U(r,\theta) = \frac{\theta^2}{2} + \ln|r| - \ln a - \frac{r}{a} + 1$, where may be taken $a = R_J$.

Now, accordance equality of (45) define force balance in the magnetosphere. As standard form define the plasma $\beta = \frac{8\pi\rho}{B^2}$ and the Alfven Mach number by

$$M_A^2 = \frac{1}{2}\beta \frac{\partial \ln \rho}{\partial \ln \omega} + \frac{4\pi}{c}\frac{I_0}{B} \tag{50}$$

In Table 1 we list the various contributions to force balance as measured in the MEP as a function of radial distance from (50)(see Appendix). It is important note that, in the tail magnetosphere of Jupiter, the Euler potential force is constant, and inside the tail, the Lorentz force $\vec{J} \times \vec{B}$ is equal to $-\rho\vec{\Omega} \times (\vec{\Omega} \times \vec{R}) - \vec{\nabla}\vec{P}$. It means that the outward centrifugal and pressure gradient forces are balanced by Lorentz force. Where $\vec{\Omega}$ is angular speed of Jupiter, $\vec{J}, \vec{B}$ current density and magnetic field, respectively; P -total thermal pressure, $\vec{R}$ -centrifugal force, ρ - cylindrical distance from the axis of rotation. Above obtained theoretical results may be checked in results of observation Voyager1-2.

6. Conclusion

In this paper considered an applications non-classical equations of mathematical physics in the fields of mechanics, astronomy, and astrophysics in the case of plasma models of the Jupiter's magnetosphere, with search of lunar Io. Given comparisons works and using observation results of Pioner-10-11, Voyager1-2 considered mathematical models. For this reason, consider physical background with bound cold plasmas and mathematical background with connected cold plasma. For applications first of all given definition of non-classical equations of mathematical physics and how to apply to the magnetosphere of Jupiter, and conditions of applicability of mathematical models in plasma models. Therefore, physical process of role of Jupiter's satellite Io, source as a hot plasma source for Jupiter was considered. By physical observation of magnetic disk acts as a balancing mechanical equilibrium to retain hot plasma was investigated. in this case was obtained derivation of the cold plasma modeling equation whose source is Jupiter's satellite Io which being to the Keldysh type equation. At first application exact solution in integrals formula. Additionally, investigated effects for corresponding geometrical analysis of the resonance curve of model equation of cold plasma and hot plasma. Analyzed the solution near singular points of the characteristics of the cold plasma equation in terminology mathematical view of points as degenerating and singularity cases. At the same time for this reason analogical physical analyses also illustrated, as cutoff, resonances for cold and hot plasmas. In the region tail of Jupiter given analyses of basic model equations of the Jupiter magnetosphere for the equilibrium between magnetic force, pressure gradient, and centrifugal force in the presence of plasma rotations. A namely, role of Alfven Mach number with constant Euler potential parameter in the region tail of Jupiter's magnetosphere agreed previously results of observation Voyager1-2.

Remark1. By the investigation we include that equilibrium between magnetic force, pressure gradient, and centrifugal force in the presence of plasma rotations proves mathematically justification magneto hydrodynamic equilibrium and steady of Jupiter rotation. For importantly estimation our investigation we reference concerning notes topics: "the author of the article "The Magnetosphere of Jupiter: Moving from Discoveries Towards Understanding," Professor Frank Crary from the University of Colorado, Boulder, Laboratory for Atmospheric and Space Physics (303) 735-2120, noted that the magnetosphere of Jupiter: from discovery to understanding: "Jupiter's magnetosphere has been observed by many spacecraft, but most of these results were discoveries of the global and general properties of the magnetosphere. They usually ask more questions than they answer. Here we present some of the remaining questions needed to truly understand Jupiter's magnetosphere and argue that this can be achieved with small, targeted missions. Despite past missions to Jupiter, many questions remain about its magnetosphere and how the various elements of this dynamic and coupled system interact. Past discoveries have shown us how much remains to be learned before we understand Jupiter's magnetosphere. How and to what extent does the solar wind control Jupiter's magnetosphere?"

Account into the Remark1, the solar wind acts as a trap for cold and hot plasma in the magnetosphere. The tail of the magnetosphere with cold plasma ensures the replacement of its hot plasma through the presence of the solar wind. But just as the loss of the temperature of the sun's surface (Parker's problem) is associated with the reconnection of magnetic force lines, such a picture is also connected with the loss of the ejection coming (almost 60 percent) from Io. The tail of the magnetosphere is from the plasma model equation, where singular degeneracy occurs and magnetic force reconnection occurs. However, due to force balance, the equilibrium of Jupiter is preserved, and the stability of magneto hydrodynamic equilibrium is consistent with mechanical equilibrium, where the Alfven Mach number is also related to the angular velocity of Jupiter. In our work, as far as possible, unresolved issues are highlighted.

Therefore, I am very grateful to Professor Frank Crary and other researchers noted, because they did not stand idly by but continued research on this topic.

Remark 2. Account into the above investigation, which has a mathematical justification for the hydrodynamic equilibrium of Jupiter, and from the author's work [54-58], which also has a mathematical justification for the hydrodynamic equilibrium of Jupiter, the proposition model of stability and rotation of Jupiter, including the astronomer's two models, proves a new justification.

Author Contributions

Mahammad A. Nurmammadov is the sole author. The author read and approved the final manuscript.

Conflicts of Interest

The author declares no conflicts of interest.

Appendix

In Table 1 we list the various contributions to force balance as measured in the MEP as a function of radial distance from (50). The initial increase of the Lorentz force with ~ followed by a decrease is indicative of the finite radial extent of the current sheet. Recent modeling has shown that a better fit to the magnetic field data can be obtained if the model current sheet is tilted toward the centrifugal equatorial plane (CEP) and such a tilt is expected if the cold component plays a significant dynamical role [53]. The weaker current sheet observed by Voyager 2 is consistent with the lack of a super—Alfvènic cold component of the plasma during that encounter, the current sheet observed at that time should have been more closely aligned with the MEP than with the CEP.

Table 1. Force balance at Jupiter.

$\bar{\omega}$	M_A	β	$M_A^2 + 1.88$	$\frac{4\pi}{\varepsilon}\frac{I_0}{B}$
17	1.7	2.4	7.2	9.0
21	1.6	5.7	13	17
25	1.6	7.4	16	20
28	1.2	4.5	9.5	24
35	1.5	1.8	9.1	28
42	1.8	1.0	5.0	25

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